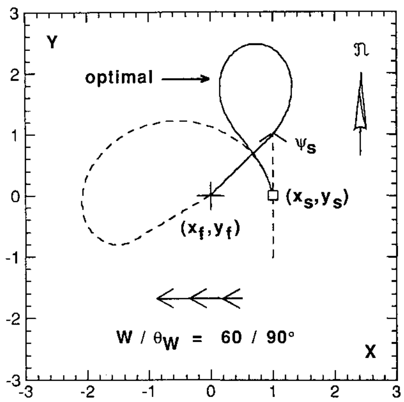

Fig. 1 $[x^*(t), y^*(t)]$, $0 \le t \le t_f$.

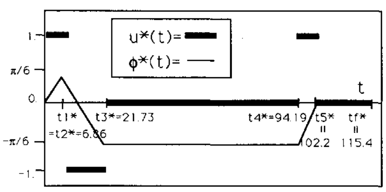

Fig. 2 $u^*(t)$, $\phi^*(t)$, $0 \le t \le t_f$, $(\alpha^* = 1)$.

wing tip of an aircraft flying due north at 200 kt with a 60-kt crosswind from the right:

Example 1:

$$(x_s, y_s),\quad (x_f, y_f),\quad V\,(\mathrm{kt}),\quad \Psi_s,\quad W\,(\mathrm{kt}),\quad \theta_W$$

Input:

$$(1, 0),\quad (0, 0),\quad 200,\quad \pi/2,\quad 60,\quad 0$$

The minimum-time trajectory for this fly-to-point objective was computed to be $\alpha^* = 1$, $t_1^* = t_2^* = 6.86$, $t_3^* = 21.73$, $t_4^* = 94.19$, $t_5^* = 102.2$, and $t_f^* = 115.4$, which is a left-right turn combination requiring 115.4 s total flight time. The optimal trajectory is given by the solid line in Fig. 1. The optimal control profile $u^*(t)$ and corresponding angle of bank history $\phi^*(t)$ are given in Fig. 2. The total clock time on a Macintosh IIci microcomputer required to compute the solution for this specific wind and fly-to-point configuration was 233 ms. Although machine language code and faster computer equipment is used in actual practice, this is already well within the time scale required for the automatic control of the maneuver. What is interesting about this example is that the computed optimal trajectory consisting of a left-right turn combination is not one that an aircrew would normally execute. Indeed, experience has shown[1] that, without additional information, the pilot in command will either choose the nonoptimal feasible solution (the dashed line in Fig. 2 that requires 136.7 s flight time) or will first execute a procedure turn into wind and away from either feasible extremal. There are many similar scenarios where the optimal trajectory is equally unintuitive.

Conclusion

In the problem considered here, an analysis of the standard necessary conditions leads to the required form for the optimal control and an efficient state-space method of solution. An important advantage of this real-time solution procedure is that the optimal trajectories can be recomputed throughout the execution of the maneuver to allow for the correction of cross-track errors (unexpected drift) and inaccurate estimates of wind speed and direction.

Acknowledgments

This research was partially supported by the Natural Sciences and Engineering Research Council of Canada under Grant A5338 and by the Canadian Department of National Defence, Defence Research Establishment Atlantic.

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U-Parameter Design for Terrain-Following Flight Control

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Introduction

THIS Note explores the application of U-parameter feedback design developed by Dorato and Li[1] to a terrain-following flight control problem. U-parameter theory permits one to optimize performance of a nominal linear time invariant system, while guaranteeing robust stability in the presence of unstructured plant perturbations. In the terrain-following problem considered here, it is assumed that a command flight path angle is computable from available radar data (see, for example, Ref. 2) so as to achieve good terrain following and that the object is to find a controller that causes the actual flight path angle to follow the command signal. The control input is taken to be the elevator angle, and the aircraft dynamics are linearized about constant nominal trajectories. Since the most significant variation in the linearized model is due to nominal flight path angle, this variable is treated as an uncertain parameter. This parameter uncertainty is transferred to an unstructured frequency domain bound on plant transfer function uncertainty. U-parameter theory is then used to minimize a norm on tracking error for nominal motion (level flight 100 m above terrain and zero flight path angle), while guaranteeing robust stability for other flight path angles. Before discussing the terrain-following problem, we briefly review U-parameter theory.

Received May 6, 1991; revision received May 13, 1992; accepted for publication May 29, 1992.

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U-Parameter Design

Kirmura[3] has shown that a compensator that stabilizes a family of plants characterized by a nominal transfer function $P_o(s)$ and an unstructured frequency domain perturbation bound $r(s)$ with property

$$|P(j\omega)-P_o(j\omega)| \le |r(j\omega)| \qquad \text{for all } \omega \tag{1}$$

where $P(j\omega)$ is the perturbed plant, can be parameterized in terms of an arbitrary strongly bounded real (SBR) function [a function $U(s)$ that is analytic for Re $s>0$ and $|u(j\omega)|<\delta<1$ for all ω]. This arbitrary function, which we refer to as the U parameter, is exploited by Dorato and Li[1] to satisfy other design specifications. For example, in this application the U parameter is used to minimize the mean square tracking error of the nominal system.

Given the data $[P_o(s), r(s)]$, U parameterization yields a controller of the form

$$C(s)=\frac{C_1(s)+C_2(s)U(s)}{C_3(s)+C_4(s)U(s)} \tag{2}$$

where $C_i(s)$ are polynomials computed from the data and $U(s)$ an arbitrary SBR function. The polynomials $C_i(s)$ may be computed from interpolation theory, as in Ref. 1, or by other, more recent, H^∞ techniques (see Ref. 4).

Terrain-Following Problem

The aircraft dynamics assumed here linearized about a nominal trajectory (level flight with 100-m ground clearance) have the following state-space representation[5]:

$$\dot{x}=Ax+Bu$$
$$y=Cx \tag{3}$$

where $x'=[v,\nu,\theta,q]$, $u=\delta_e$, and $y=\nu$; with $v=$ flight velocity (m/s), $\nu=$ flight path angle (rad), $\theta=$ pitch rate (rad/s), $q=$ pitch angle (rad), and $\delta_c=$ elevator angle (rad); and

$$A=\begin{bmatrix} -0.0097 & 0.0016 & 0 & a14 \\ 0.0955 & -1.4300 & 0 & a24 \\ -1.4751 & 14.2831 & -2.7784 & a34 \\ 0 & 0 & 1.0000 & 0 \end{bmatrix}$$

$$B=\begin{bmatrix} 0 \\ 0.1095 \\ -26.006 \\ 0 \end{bmatrix}, \qquad C'=\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

The nominal flight path angle ν_o is assumed to range over [−30, 30 deg], and the values of $a14$, $a24$, and $a34$ are given by: $a14=-0.0016-0.0484\cos\nu_o$, $a24=1.43+0.0488\sin\nu_o$, and $a34=14.2831-0.0488\sin\nu_o$.

To keep the controller as simple as possible, the value of $r(s)$ is selected to be a constant, given by

$$r(s)=\sup_{\omega}\sup_{\nu_o}|P(j\omega)-P_o(j\omega)| \tag{4}$$

where $P_o(j\omega)$ is the aircraft transfer function between input δ_e and output ν when $\nu_o=0$. With actuator dynamics $-1/(0.1s+1)$ and precompensator given by

$$C_p(s)=\frac{(s+0.0477)(s+2.106\pm 3.7193j)}{(s+0.0064)(s+19.481)(s+3.182\pm 3.182j)} \tag{5}$$

the nominal plant ($\nu_o=0$) becomes

$$P_o(s)=\frac{-1.0950(s-16.7)}{(s+10)(s+3.182\pm 3.182j)(s-0.0417)} \tag{6}$$

The precompensator is used to simplify $P_o(s)$ as much as possible and to provide a short-period damping ratio of 0.4927 to meet flying specification,[6] without causing any unstable pole/zero cancellation. We assume here that cancellation of stable modes is close enough so that undesired stable modes in the transient response have small amplitude.

With $P_o(s)$ given by Eq. (6) and ν_o in the range [−30, 30 deg], we compute $r(s)$ from Eq. (4) to be $r(s)=0.0280$.

Design Objectives

The design objectives here are twofold:

1) Robust stability: The linearized closed-loop system is to be stable for all ν_o in the range [−30, 30 deg].

2) Nominal performance: The steady-state tracking error to a step-input flight path command signal, when $\nu_o=0$, should be zero, and the mean square tracking error should be as small as possible.

U-Parameter Solution

Using the interpolation theory in Ref. 1 and the given data, i.e., $r(s)=0.0280$ and $P_0(s)$ given by Eq. (6), the compensator polynomials $C_i(s)$ required for robust stability are computed to be

$$C_1(s){:}=[0.3163, 4.5103, 19.3160, 58.703, 2.7234, -0.1021, -4.7942\times 10^{-3}]$$
$$C_2(s){:}=[-1, -14.176, -59.884, -180.55, 6.6577, 0.4042, -1.5157\times 10^{-2}]$$
$$C_3(s){:}=[0.0028, 1.005, 16.7, 52.952, 0.3033, -9.2793\times 10^{-2}, -5.9244\times 10^{-4}]$$
$$C_4(s){:}=[-8.8564\times 10^{-3}, -0.3175, -4.650, -19.148, 321.98, -11.337, -8.5749\times 10^{-2}] \tag{7}$$

where $a_n s^n + a_{n-1}s^{n-1} + \cdots + a_0 \equiv [a_n, a_{n-1}, \ldots, a_0]$.

The U parameter in Eq. (2) is then used to minimize the weighted mean square tracking error (H^2 norm):

$$J=\left(\int_{-\infty}^{\infty}|W(j\omega)S(j\omega)|^2\,d\omega\right)^{1/2}=\|W(s)S(s)\|_2 \tag{8}$$

where the weighting factor $W(s)$ is taken to be $W(s)=1/s$ to enforce a zero steady-state error and $S(s)=[1+C(s)P_o(s)]^{-1}$.

There is currently no known way to analytically minimize J with respect to $U(s)$. However, with further parameterization of $U(s)$, a numerical solution can be obtained for a particular problem. We select a very simple parameterization of $U(s)$ given by

$$U(s)=\frac{as+b}{s+1} \tag{9}$$

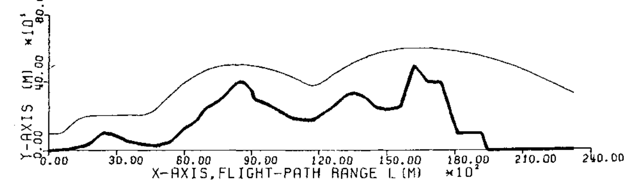

Fig. 1 Simulated terrain-following path.

with $b = -C_3(0)/C_4(0) = -0.0069$ to guarantee a zero steady-state error, and we use a to minimize J, subjected to the constant that $|a| < 1$, in order that $U(s)$ remain SBR. By simple discretization, the optimal value of a is computed to be $a = -0.07$. With the value of a, the compensator is computed from Eqs. (9), (7), and (2).

Figure 1 shows a simulated flight path for a typical terrain contour.

Conclusion

U-parameter theory is presented as a design tool for a terrain-following flight control problem. The results are based on a linearized uncertain model of the actual nonlinear dynamics; however, simulation results show that fairly good terrain-following characteristics can be obtained with this design approach. Even though the U-parameter theory applied to this example is polynomially based and limited to single-input/single-output systems, it is possible, with recent results in H^∞ control,[4] to do U parameterization for multivariable systems using only matrix computations.

Acknowledgments

This research was supported by the China National Science Foundation under Grant 68974023 and the U.S. National Science Foundation under Grant INT-8821843.

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Exact Conversion of Earth-Centered, Earth-Fixed Coordinates to Geodetic Coordinates

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I. Introduction

THE transformations between Earth-centered, Earth-fixed (ECEF) coordinates and geodetic coordinates are required in many applications—for example, in NAVSTAR/GPS navigation and geodesy. The exact transformation from geodetic coordinates to ECEF coordinates is well known,[1] but the exact inverse transformation (from ECEF directly to geodetic) is still unavailable in open literature, though both approximation methods[1-3] and an indirect method[4] do exist for converting ECEF coordinates to geodetic coordinates. In this Note, the formulas of exact transformation from ECEF coordinates to geodetic coordinates are developed in closed form.

Received Sept. 18, 1991; revision received April 20, 1992; accepted for publication April 21, 1992.

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II. Mathematical Formulation

We are going to determine the geodetic coordinates (ϕ,λ,h) of point P as a function of its ECEF coordinates (x,y,z). Here,

ϕ = geodetic latitude (positive north)
λ = geodetic longitude (measured east from the Greenwich meridian)
h = altitude normal to ellipsoid
a = ellipsoidal equatorial radius ($a = 6378.137$ km for model WGS-84)
e = eccentricity of ellipsoid ($e^2 = 0.00669437999$ for model WGS-84)
b = ellipsoidal polar radius ($b = a\sqrt{1-e^2}$)

The geodetic longitude λ can be determined in four quadrants by the identity

$$\lambda = 2\arctan\left[\left(\sqrt{x^2+y^2}-x\right)/y\right] \tag{1}$$

Now consider the meridian plane of point P and let $w = \sqrt{x^2+y^2}$. Suppose $Q(u,v)$ is an arbitrary point in this meridian plane satisfying

$$\frac{u^2}{a^2} + \frac{v^2}{b^2} = 1 \tag{2}$$

and $P_1(w_1,z_1)$ is the subpoint of $P(w,z)$ on the ellipsoid. By noting that altitude h is the shortest distance from $P(w,z)$ to the ellipsoid, we have $h = f(w_1,z_1) \le f(u,v)$ for all points $Q(u,v)$, where $f(u,v)$ is defined to be the distance from $P(w,z)$ to $Q(u,v)$, i.e.:

$$f(u,v) = \sqrt{(u-w)^2 + (v-z)^2}$$

The minimum point $P_1(w_1,z_1)$ can be determined by Lagrange's method. Let

$$g(u,v) = f^2(u,v) + \theta\left(\frac{u^2}{a^2} + \frac{v^2}{b^2} - 1\right)$$

The conditions

$$\frac{\partial g(w_1,z_1)}{\partial u} = 2(w_1 - w) + \theta\frac{2w_1}{a^2} = 0$$

and

$$\frac{\partial g(w_1,z_1)}{\partial v} = 2(z_1 - z) + \theta\frac{2z_1}{b^2} = 0$$

generate

$$w_1 = \frac{w}{1+\theta/a^2}, \qquad z_1 = \frac{z}{1+\theta/b^2} \tag{3}$$

Now we have

$$\left(\frac{w/a}{1+\theta/a^2}\right)^2 + \left(\frac{z/b}{1+\theta/b^2}\right)^2 = 1 \tag{4}$$

since (w_1,z_1) satisfies Eq. (2). From Eq. (3) we have $\theta > -b^2$. Noting that $e^2 = 1-(b/a)^2$ and letting $l = e^2/2$, $m = (w/a)^2$, and $n = [(1-e^2)z/b]^2$, we can substitute $\theta = a^2(t-1+l)$ into Eq. (4) to get the equation

$$\frac{m}{(t+l)^2} + \frac{n}{(t-l)^2} = 1 \tag{5}$$